EVALUATION OF SOME SELECTION DESIGNS BEFORE
MATING FOR OFFSPRING FROM FAMILIES OF FIXED SIZE

P.J. Nicholls

Department of Agriculture,
Rawson Place, Sydney, New South Wales 2000

THE SELECTION DIFFERENTIAL CONSIDERED BEFORE MATING

For the improvement of desirable animal traits, directional selection
is generally used to obtain a desired response. To the extent that
average response is proportional to the phenotypic selection
differential, the consequence of a selection design can be projected
from the properties of the selection differential. Before selecting
members of one generation as parents for the next, it is usual to
consider alternative selection designs and assess their anticipated
responses from the expected selection differentials. For the case
of single offspring when r are to be selected from a random sample
of n, the expected selection differential $k_{r,n}$ may be compared for
various combinations of r and n.

A potentially wider scope of alternative designs can be considered
by projecting the consequences of selection from parents of the
selected generation. This involves a distribution of offspring sample
size and, when the parameters of the distribution are known, it is
possible to evaluate the expected selection differential conditional
on parent sample size S. By manipulating S and r, alternative
selection designs can be evaluated before mating.

When offspring sample size is a random variable the selection
differential depends on the selection procedure followed. For a
breeding unit in which older animals are cast for age the number of
parents to be replaced may be known before mating: with unconditional
replacement r are selected from n offspring when r < n, otherwise all
n are used; alternatively replacement may be conditional on a minimum
offspring sample size or on achieving a minimum selection intensity.

When S parents are chosen randomly and each has the same probability
p of contributing a family of fixed size m with the desired trait,
the resulting distribution of families is binomial

$$Pr(s \text{ families}) = Pr(ms \text{ offspring}) = \binom{S}{s} P^s(1-p)^{S-s}, \quad s=0,1,\ldots,S$$

so that the offspring sample has at most N=mS members and has expected
value Np = mSp. Of course, N=S for m=1.

The expected selection differential $k_{r,N,p}$ for the case m=1 was considered for a normal trait by Nicholls (1984). For m>1 the presence of related individuals induces correlations within families: it will be assumed a common intraclass correlation t, $-1/(m-1) < t < 1$, exists within families, with zero correlation among families. The expected selection differential before mating $k_{r,N,p}(m,t)$ can then be considered (Nicholls 1982). For evaluations in the case of a normal trait, numerical values of $k_{r,n}(m,t)$ obtained by Hill (1976) for n up to 40 or 42 and m up to 8 are useful.

THE EFFECTS OF m, t AND p ON $k_{r,N,p}(m,t)$

Given N, r and p, the effects of group size m and intraclass correlation t on $k_{r,N,p}(m,t)$ for the normal distribution depend largely on the size of r relative to Np. When r is small relative to Np, $k_{r,N,p}(m,t)$ decreases as either m or t increases, and at an increasing rate for each. When N is large relative to m the decrease with t is not great, but is much more marked at lower N, that is, when the likely number of groups is small with a high average pairwise correlation. Examples of these effects are shown below.

N	r	p	t	m=2	3	4	6
12	1	0.8	0.0	1.502	1.492	1.480	1.448
			0.5	1.430	1.354	1.284	1.164
			0.9	1.281	1.103	0.956	0.735
36	3	0.8	0.0	1.650	1.648	1.645	1.639
			0.5	1.628	1.604	1.579	1.529
			0.9	1.591	1.538	1.463	1.326

However as r increases relative to Np, $k_{r,N,p}(m,t)$ is affected by the relative sizes of r and m and does not necessarily decrease with larger m, but the decline with increasing t remains, although at a reduced rate, as shown below.

N	r	p	t	m=2	3	4	6
12	4	0.4	0.0	0.269	0.316	0.274	0.392
			0.5	0.257	0.288	0.246	0.304
			0.9	0.246	0.250	0.223	0.186

Considering the trend of $k_{r,N,p}(m,t)$ with increasing p, the table below shows that the rate of increase when p is low decreases for larger m and this effect is more pronounced at higher t, particularly when r is small relative to Np. However, as p nears 1 the rate of increase is little affected by m or t unless t is high and r is large relative to N, when the rate of increase is reduced. Given t, the

difference $k_{r,N,p}(m,0) - k_{r,N,p}(m,t)$ is fairly constant when p is high compared to r/N and the difference increases for larger m, particularly when r/N is low.

N	m	r	t	p=0.2	0.5	0.7	0.9	1.0
12	2	1	0.0	0.603	1.193	1.420	1.571	1.629
			0.9	0.354	0.930	1.187	1.359	1.426
		4	0.0	0.052	0.419	0.713	0.936	1.019
			0.9	0.047	0.385	0.665	0.883	0.965
	6	1	0.0	0.471	1.041	1.330	1.548	1.629
			0.9	0.166	0.437	0.633	0.840	0.948
		4	0.0	0.193	0.493	0.700	0.911	1.019
			0.9	0.077	0.253	0.412	0.604	0.712

When assessing the influence of intraclass correlations on the selection differential before mating, Nicholls (1982) found that compared to the case for t=0.0, a correlation t=0.5 can lead to a reduction in expected values that is not insignificant, particularly when p is much less than 1.0 and relatively few offspring are to be chosen from a small number of groups. This contrasts with the finding of Hill (1976) that $k_{r,n}(m,t)$ is little affected by t for values likely to be found in practice (t<0.5) when individuals are selected, which led him to conclude that t could be ignored in practice.

AN APPROXIMATION FOR $k_{r,N,p}(m,t)$

An approximation for $k_{r,N,p}(m,t)$ based on the expected offspring sample size Np is

$$k_{r,N,p}(m,t) \simeq (1-c_a)^{1/2} k_{r,Np}$$

where $k_{r,Np}$ is the expected selection differential when r are chosen from a random sample of size Np and $c_a = (m-1)t/(Np-1)$ is the average pairwise correlation of a sample of size Np where families of m occur with intraclass correlation t. Unless m is large or r is close to Np the approximation is satisfactory when t is low. However for high t the approximation is generally poorer when r is small and m large relative to Np. Provided p is not low the approximation should be satisfactory for N>40 unless selection is likely to consist of choosing relatively few offspring from a small number of groups with a high average pairwise correlation.

REFERENCES

Hill, W.G. (1976) Biometrics 32 : 889-902
Nicholls, P.J. (1982) M.Sc. Thesis, U.N.S.W.
Nicholls, P.J. (1984) Proc. AAABG 4th Conf. Adelaide 4-6 June p.12-13